AFCRL-66-142

# SOME CODES WHICH ARE INVARIENT UNDER A DOUBLY-TRANSITIVE PERMUTATION GROUP AND THEIR CONNECTION WITH BALANCED INCOMPLETE BLOCK DESIGNS



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Code 1

CONTRACT NO. AF19(628)-4379
PROJECT NO. 5628
TASK NO. 562801

SCIENTIFIC REPORT NO. 6

JANUARY 28, 1966

PREPARED FOR
AIR FORCE CAMBRIDGE RESEARCH LABS.
OFFICE OF AEROSPACE RESEARCH
UNITED STATES AIR FORCE
BEDFORD, MASSACHUSETTS

DEPARTMENT OF
ELECTRICAL ENGINEERING
UNIVERSITY OF HAWAII
HONOLULU, HAWAII

AFCRL - 66 - 142

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Air Force Cambridge Research Laboratories
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## ABSTRACT

If a binary code is invariant under a doubly-transitive permutation group, then the set of all code words of weight j forms a balanced incomplete block design. Besides the extended normal BCH codes and the extended quadratic residue codes, the Reed-Muller codes are proven to be invariant under a doubly-transitive permutation group. Thus, BIB designs can be derived from these classes of codes. It is shown that if the symbols of the Reed-Muller codes are properly arranged, and if the first digit is omitted, then all Reed-Muller codes are cyclic.

# SOME CODES WHICH ARE INVARIANT UNDER A DOUBLY-TRANSITIVE PERMUTATION GROUP AND THEIR CONNECTION WITH BALANCED INCOMPLETE BLOCK DESIGNS

(T. Kasami and S. Lin)

## 1. Introduction

Two classes of codes which are invariant under a doubly-transitive affine group of permutations will be presented in this report. It is our objective to show a connection between these classes of codes and some balanced incomplete block designs. One class of these codes is the extended normal Bose-Chaudhuri-Hocquenghem codes which have been proven to be invariant under a doubly-transitive affine group of permutations by W. W. Peterson ( 1 ). In this report, we shall prove that Reed-Muller codes also have this invariant property.

For convenient reference, the definition of a balanced incomplete block design is presented here.

Definition 1-1: A balanced incomplete block design is an arrangement of v objects into b sets satisfying the following conditions:

(1) Each set contains exactly k different objects.

(2) Each object occurs in exactly r different sets.

(3) Any pair of objects occurs in exactly $\lambda$ different sets.

The integers v, b, r, k, $\lambda$ are called the parameters of the design and it is easy to show that they satisfy the following relations

$$bk = vr \tag{1}$$

$$\lambda(v-1) = r(k - 1) \tag{2}$$

We also call a balanced incomplete block design as a $(b,v,r,k,\lambda)$ - configuration ( 5 ). An important area of application for block designs is in the design of experiments for statistical studies. For a thorough treatment of this subject the reader is referred to Henry B. Mann ( 3 ) and R. C. Bose ( 6, 8 ). In consequence of this use the objects are called "treatments" and the sets are called "blocks". With any block design, we may associate an incidence matrix A.

Definition 1-2: If S is a block design with blocks $B_1, B_2, \ldots B_b$ and objects $O_1, O_2, \ldots O_v$, the incidence matrix $A = (a_{ij})$, where $i = 1, 2, \ldots, b$ and $j = 1, 2, \ldots v$, of S is defined by the rules

$$\begin{aligned} a_{ij} &= 1 \qquad \text{if } O_j \in B_i \\ a_{ij} &= 0 \qquad \text{if } O_j \notin B_i \end{aligned} \tag{3}$$

Clearly, A is a bxv matrix of zeros and ones such that each row contains k ones, each column contains r ones, and any two columns have ones in corresponding positions exactly $\lambda$ times. Conversely, the existence of a matrix A with these properties is equivalent to the existence of a block design.

In this report, we shall show that some block designs can be derived from the well known Bose-Chaudhuri-Hocquenghem codes and Reed-Muller Codes. Both of these two classes of codes are invariant under a doubly-transitive affine group of permutations.

## 2. Codes which are invariant under a doubly-transitive affine group of permutations and their connection with balanced incomplete block designs

Let $\alpha$ be a primitive element of $GF(2^m)$. A t-error correcting binary BCH code is obtained by requiring

$$\alpha, \alpha^2, \alpha^3, \ldots, \alpha^{2t} \tag{4}$$

to be roots of any code vector f(x). Then an extended normal BCH code is defined as follows;

Let V be the code which is the null space of the matrix

$$H = \begin{bmatrix} 1 & 1 & 1 & 1 & . & . & . & . & 1 \\ 0 & 1 & \alpha & \alpha^2 & & & & & \alpha^{n-1} \\ 0 & 1 & \alpha^2 & \alpha^4 & & & & & \alpha^{2(n-1)} \\ . & & & & & & & & \\ . & & & & & & & & \\ . & & & & & & & & \\ 0 & 1 & \alpha^{2t} & \alpha^{4t} & . & . & . & . & \alpha^{2t(n-1)} \end{bmatrix} \tag{5}$$

where $n = 2^m-1$. This is a normal BCH code with an overall parity check added as the first digit. This class of codes have been proven to be invariant under a doubly-transitive affine group of permutations ( 1 ) by W. W. Peterson. We shall state the theorem without proof.

Theorem 2 - 1 (Peterson): An extended normal BCH code is invariant under a doubly-transitive affine group of permutations of its digits. More specifically, let us number each digit in a code vector with the Galois field element that appears in the corresponding position in the second row of H, i.e. the first symbol in each code vector is numbered o, and for i, the ith digit is numbered as $\alpha^{i-2}$. The theorem states that for any field elements a and b, if for any code vector, the symbol in position X is permuted to position aX + b, the resulting vector is also a code vector in V.

Next, we shall prove that Reed-Muller codes are also invariant under a doubly-transitive affine group of permutations. Let us construct a matrix as follows;

$$H = \begin{bmatrix} 1 & 1 & 1 & 1 & . & . & . & 1 \\ 0 & 1 & \alpha & \alpha^2 & & & & \alpha^{n-1} \end{bmatrix} \tag{6}$$

where $\alpha$ is a primitive element of $GF(2^m)$ and $n = 2^m-1$. The null space V of H is an extended binary NBCH code (or extended Hamming code). The field $GF(2^m)$ can be considered as a vector space of dimension m over the binary field. If each $\alpha^i$ in H is represented as a column vector of m binary digits, H is then a $(m + 1)X2^m$ matrix

$$H = \begin{bmatrix} v_o \\ v_1 \\ v_2 \\ \cdot \\ \cdot \\ \cdot \\ v_m \end{bmatrix} \tag{7}$$

where $v_o = (1, 1, 1, . . . , 1)$. H is just the generator matrix of the first order Reed-Muller code of length $2^m$ with different permutation of columns. The row space U of H is the first order Reed-Muller code and is the null space of V which is a special case of the extended NBCH codes. Since V is invariant under a doubly-transitive affine group of permutations, U is also, by the following theorem.

Theorem 2-2 ( 2 ): If V is invariant under a group G of permutations P, then the null space U of V is also invariant under G.

Now, let us define the vector product of two vectors as follows:

$$\begin{aligned} u &= (a_1, a_2, a_3, . . . , a_n) \\ v &= (b_1, b_2, b_3, . . . , b_n) \\ uv &= (a_1b_1, a_2b_2, a_3b_3, . . . , a_nb_n) \end{aligned} \tag{8}$$

Then, the rth order Reed-Muller code is formed by using as a basis the vectors $v_o, v_1, v_2, . . . , v_m$ of H of Eq. (7) and all vector products of these vectors r or fewer at a time, where $r \leq m$. Therefore, each code vector of the rth order Reed-Muller code is a linear combination of vectors $v_o, v_1, v_2, \ldots, v_m$ and their vector products. We can express every code vector as a polynomial of degree r or less in $v_o, v_1, v_2, \ldots, v_m$

$$w = \Gamma(v_o, v_1, v_2, \ldots, v_m) \tag{9}$$

We know that the first order Reed-Muller code is a subcode of the rth-order Reed-Muller code and is invariant under a doubly-transitive affine group G of permutations P. $Pv_j = v_j' = \sum_{i=0}^{m} a_i v_i$ is a linear combination of $v_o, v_1, v_2, \ldots, v_m$, where $j = 0, 1, 2, \ldots, m$. $P(w) = P\Gamma(v_o, v_1, v_2, \ldots, v_m) = \Gamma(Pv_o, Pv_1, Pv_2, \ldots, Pv_m)$. When $\Gamma(w) = (Pv_o, Pv_1, \ldots, Pv_m)$ is expanded, this results in a different polynomial $\Gamma'(v_o, v_1, v_2, \ldots, v_m)$ of degree r or less in $v_o, v_1, v_2, \ldots v_m$. Thus $\Gamma'(v_o, v_1, v_2, \ldots, v_m)$ is also a codevector of the rth order Reed-Muller code. This implies that the rth order Reed-Muller code is invariant under a doubly-transitive affine group of permutations. Let Q be any permutation, then $V' = QV$ is an equivalent code of V. It is easy to show that, if code V is invariant under a group G of Permutations P, then V' is invariant under the group G' of permutations $Q^{-1}PQ$ with $P \in G$. Thus, the Reed-Muller codes in original form are also invariant under a doubly-transitive group of permutations.

Let $\alpha$ be a primitive element of $GF(2^m)$. Consider a Reed-Muller code of any order (in the form described in this report) and the permutation $\alpha X$. This permutation will leave the first digit of every code vector unpermuted, but shift cyclically the rest $2^m$-1 digits by one position. Since the code is invariant under the permutation $\alpha X$, thus, the shortened code obtained by deleting the first digit of each code vector of the Reed-Muller code is cyclic.

Suppose that the code V is invariant under a doubly-transitive permutation group and let $S_k$ be the set of codevectors of weight k. It is easy to see that $S_k$ is also invariant under the doubly-transitive permutation group. Thus, we have the following two theorems.

Theorem 2-3. Let N(k) be the number of code vectors of weight k of a code V which is invariant under a doubly-transitive permutation group. If these code vectors are arranged as a N(k)xn matrix A, then the number of ones in each column of A is constant and is equal to

$$r = \frac{k\ N(k)}{n} \tag{10}$$

where n is the length of the code V.

Proof. Because the permutation group is transitive, for every $\ell$, there exists a permutation that carries column 1 into column $\ell$. This permutation leaves the rows of A unchanged except it rearranges the rows. It follows that column 1 and column $\ell$ of A have the same number of ones which is equal to

$$r = \frac{k\ N(k)}{n} \tag{11}$$

Theorem 2-4. Any two columns of the matrix A have ones in corresponding positions exactly $\lambda$ times, and $\lambda$ is equal to

$$\lambda = \frac{k(k-1)N(k)}{n(n-1)} \tag{12}$$

Proof. Because the permutation group is doubly-transitive, there exists a permutation which will permute the ith column of A to the first column of A, and the jth column to the second column. Since the permutation permutation leaves the rows of A invariant except it rearranges the rows, we can rearrange the rows to obtain the original matrix A. It follows that the number of ones in corresponding positions between ith and jth columns is exactly equal to the number of ones in corresponding positions between the first and second columns. This implies the theorem. It is easy to show that

$$\lambda = \frac{k(k - 1)N(k)}{n(n - 1)} \tag{13}$$

The matrix A is thus a N(k)xn matrix with zeros and ones, such that each row has exactly k ones, each column has exactly $r = \frac{kN(k)}{n}$ ones, and any two columns have ones in corresponding positions for exactly $\lambda = \frac{r(k-1)}{n-1}$ positions. Therefore, A is the incidence matrix of a balanced incomplete block design with parameters.

$$v = n, \qquad b = N(k), \qquad r = \frac{kN(k)}{n}$$

$$k, \text{ and } \quad \lambda = \frac{r(k - 1)}{n} \tag{14}$$

From theorem 2-3 and theorem 2-4, we have observed that if a binary code is invariant under a doubly-transitive permutation group G, then the set of all code vectors of weight k forms a balanced incomplete block design. In this report, we have shown that extended binary BCH codes of length 2 and Reed-Muller codes are invariant under a doubly-transitive permutation group, thus BIB designs can be derived from them. The extended quadratic residue codes have also been proven to be invariant under a doubly-transitive permutation group ( 9 ). The connection between perfect codes and steiner systems has been discussed by Assmus and Mattson (7).

## 3. Examples

Example 1. Consider the extended 2-error correcting NBCH code with $m = 4$. The weight distribution is as follows:

| Weight | Number of code vectors |
|---|---|
| 0 | 1 |
| 6 | 48 |
| 8 | 30 |
| 10 | 48 |
| 16 | 1 |

Three BIB designs can be designs can be derived from this code with parameters as follows:

(a) $v = 16$ $b = 48$ $r = 18$, $k = 6$, $\lambda = 6$.

(b) $v = 16$, $b = 30$, $r = 15$, $k = 8$, $\lambda = 7$.

(c) $v = 16$, $b = 48$, $r = 30$, $k = 10$, $\lambda = 18$.

Example 2. Consider the extended NBCH 5-error correcting code with $m=5$. The weight distribution is

| | |
|---|---|
| 0 | 1 |
| 12 | 496 |
| 16 | 1054 |
| 20 | 496 |
| 32 | 1 |

Three BIB designs can be derived from this code with parameters as follows

(a) $v = 32$, $b = 496$, $r = 186$, $k = 12$, $\lambda = 66$.

(b) $v = 32$, $b = 1054$, $r = 527$, $k = 16$, $\lambda = 255$.

(c) $v = 32$, $b = 496$, $r = 310$, $k = 20$, $\lambda = 190$.

Example 3. Consider the second order Reed-Muller code with m=4. The basis vectors of this code are.

| | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $v_o$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| $v_1$ | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 1 | 1 | 1 |
| $v_2$ | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 1 | 1 | 1 | 1 | 0 | 0 |
| $v_3$ | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 1 | 1 | 1 | 1 | 0 |
| $v_4$ | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 1 | 1 | 1 | 1 |
| $v_1v_2$ | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 1 | 0 | 0 |
| $v_1v_3$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 1 | 0 |
| $v_1v_4$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |
| $v_2v_3$ | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 |
| $v_2v_4$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 1 | 1 | 0 | 0 |
| $v_3v_4$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 |

The weight distribution of this code is as follows:

| | |
|---|---|
| 0 | 1 |
| 4 | 140 |
| 6 | 448 |
| 8 | 870 |
| 10 | 448 |
| 12 | 140 |
| 16 | 1 |

5 block designs can be derived from this code, they are

(a) $v = 16$, $b = 140$, $r = 35$, $k = 4$, $\lambda = 7$.

(b) $v = 16$, $b = 448$, $r = 168$, $k = 6$, $\lambda = 56$.

(c) $v = 16$, $b = 870$, $r = 435$, $k = 8$, $\lambda = 203$.

(d) $v = 16$, $b = 448$, $r = 280$, $k = 10$, $\lambda = 168$.

(e) $v = 16$, $b = 140$, $r = 105$, $k = 12$, $\lambda = 77$.

Example 4. Consider the extended quadratic residue (17, 8) code with weight distribution

| | |
|---|---|
| 0 | 1 |
| 6 | 6 X 17 |
| 8 | 9 X 17 |
| 10 | 9 X 17 |
| 12 | 6 X 17 |
| 18 | 1 |

Four block designs can be derived from this code with parameters as follows:

(a) $v = 18$, $b = 102$, $r = 34$, $k = 6$, $\lambda = 10$.

(b) $v = 18$, $b = 153$, $r = 68$, $k = 8$, $\lambda = 28$.

(c) $v = 18$, $b = 153$, $r = 85$, $k = 10$, $\lambda = 45$.

(d) $v = 18$, $b = 102$, $r = 68$, $k = 12$, $\lambda = 44$.

## 4. Conclusion.

We have shown that if a binary code is invariant under a doubly-transitive permutation group, then the set of all code vectors of weight k forms the incidence matrix of a balanced incomplete block design. We have also proven that the extended binary normal BCH codes and the Reed-Muller codes are invariant under a doubly-transitive permutation group, thus BIB designs can be derived from them. We have also observed that if the symbols of the Reed-Muller codes are properly arranged, and if the first digit is omitted, then all Reed-Muller codes are cyclic. Thus, the generator polynomial of a shortened Reed-Muller code may be found. With the generator polynomial and the invariant property under a doubly-transitive permutation group, the encoding and the decoding procedures for a Reed-Muller code may be simplified. Also, it seems that Reed-Muller codes might be invariant under a tribly-transitive permutation group.

## ACKNOWLEDGMENT

The authors wish to thank Professor W. W. Peterson of the Electrical Engineering Department, University of Hawaii, for his help and comments.

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| | |
|---|---|
| 1. ORIGINATING ACTIVITY *(Corporate author)*<br>Department of Electrical Engineering<br>University of Hawaii<br>Honolulu, Hawaii | 2a. REPORT SECURITY CLASSIFICATION<br>Unclassified<br>2b. GROUP |

3. REPORT TITLE
SOME CODES WHICH ARE INVARIANT UNDER A DOUBLY-TRANSITIVE PERMUTATION GROUP AND THEIR CONNECTION WITH BALANCED INCOMPLETE BLOCK DESIGNS

4. DESCRIPTIVE NOTES *(Type of report and inclusive dates)*
Scientific Report No. 6

5. AUTHOR(S) *(Last name, first name, initial)*
Kasami, Tadao
Lin, Shu

| | | |
|---|---|---|
| 6. REPORT DATE<br>January 28, 1966 | 7a. TOTAL NO. OF PAGES<br>13 | 7b. NO. OF REFS<br>9 |
| 8a. CONTRACT OR GRANT NO.<br>AF19(628)-4379<br>b. PROJECT AND TASK NO.<br>5628-01<br>c. DOD ELEMENT 61445014<br>d. DOD SUBELEMENT 681305 | 9a. ORIGINATOR'S REPORT NUMBER(S)<br>6<br>9b. OTHER REPORT NO(S) *(Any other numbers that may be assigned this report)*<br>AFCRL - 66 - 142 | |

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13. ABSTRACT

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| 14. KEY WORDS | LINK A ROLE | LINK A WT | LINK B ROLE | LINK B WT | LINK C ROLE | LINK C WT |
|---|---|---|---|---|---|---|
| Doubly-transitive permutation group | 8 | 3 | | | | |
| Balanced incomplete block design | 8 | 3 | | | | |
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